

Investigation 5

Magic squares

Can you complete these magic squares?
Be careful, some are harder than others

8		9
	6	
3		4

13	9	8
12		

3		
10	5	
2		

Each row, column and diagonal must add up to the same number.

	0	
	5	
2		3

6		11
7		12

7		12
8		13

	-1	
	4	
	9	2

9		
8		6
		5

You must not use the same number twice in any magic square.

Extension
Use the last square to make your own magic square.

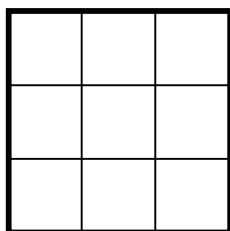
Here is a way of making a magic square

Step One

Record the number of different ways that you can make 15 by using any three of the numbers from 1-9. Each number may only appear once in each triplet of numbers

Example; $3 + 7 + 5$
 $2 + 5 + 8$

Here are two triplets, how many more are there?



Step two

Notice that some numbers in your triplets appear more often than others. Think about where you will begin to place them on a magic square.

Which number appears the most? Where will you place it?
Which numbers will be the corner numbers?

- How many possible magic squares can you make using this method?
- Do you notice a relationship between all of the magic squares that you have made?

Extension

Investigate: Is possible to create a magic square using any nine different consecutive numbers?

Learning Objectives linked to the N.N.S

Add several numbers

Solve mathematical problems or puzzles and explain patterns and relationships, generalise and predict....

Magic Squares

This is the traditional magic square. The columns, rows, and diagonals will add up to the same number, the magic number, in this case 15.

What do you notice about;

- the position of the numbers
- the relationship of the central numbers to the magic number
- the relationship of the central number to the numbers on either side
- the corner numbers
- the middle numbers of the rows and columns
- the total of all the numbers of the magic square and the number 5
- the relationship between the total of all the numbers and the sum of each line?

8	1	6
3	5	7
4	9	2

Ways of generating more magic squares

Supposing each number in the magic square was altered in the same way, what would happen? Example: What will happen when 1 is added to each number?

How many different operations can you use?

Extension: Make a magic square with negative numbers

Try making a magic square using fractions or decimals

Learning Objectives linked to the N.N.S.

Recognise and order negative numbers

Recognise odd and even numbers and make general statements about them

Recognise multiples

Find fractions of numbers or quantities

Know with rapid recall addition and subtraction facts

An alternative method for generating magic squares

Here is an alternative way of generating magic squares. Start with any number you wish and place it in the top left square on the grid. e.g. 3. Then decide how much you will add horizontally and vertically - in this example we have chosen +5 and +6

Step One

3		

Step Two

3	8	13
9	14	19
15	20	25

+5 →

↓
+6

Step Three

Here are the numbers made taken in order: 3, 8, 13, 9, 14, 19, 15, 20, 25. Now use the traditional magic square and replace the 1 with our first number, the 2 with our second number and so on until the new magic square is complete.

8	1	6
3	5	7
4	9	2

20	3	19
13	14	15
9	25	8

Investigate

Use the above method to make some magic squares. What would happen if instead of adding numbers horizontally and vertically, I subtract or even mix the two operations?

Would it work if I multiply?

Creating a formula for a magic square

Take any magic square that you have made so far. As an example we will use the traditional magic square which uses the numbers 1 - 9.

8	1	6
3	5	7
4	9	2

Diagram 1

$n + 3$		
	n	

Diagram 2

Supposing that we substitute the central number with the algebraic symbol 'n' which has any value. We then look at the relationship of each number in the square to the symbol 'n'. For example; since 8 is 5 more than the central number we will call it ' $n + 5$ '. Now can you complete the rest of the values in the square?

Investigate

You are now ready to generate as many magic squares as you wish. Draw a magic square and make 'n' any number you wish. Now use the formula that you have made to complete the magic square. Make some more magic squares using different starting numbers.

Extension

What happens if we make 'n' any other number on the square? Does the formula still work?

Magic Squares

N.B. Before attempting this investigation the children should have completed Investigation 9

Look at this formula for a magic square

$n+16$	$n-21$	$n+5$
$n-11$	n	$n+11$
$n-5$	$n+21$	$n-16$

Add up the values of the rows and columns
What do you notice?

Now compare formulae for magic squares
that you and your friends have made - what
do you notice?

What do you notice about the values of:
the corners
the side numbers?

$n+2$	$n-7$	$n+5$
$n+3$	n	$n-3$
$n-5$	$n+7$	$n+2$

Is this formula
correct?
How can you
tell?

Extension

Use the idea of a formula and play around with the values of the numbers around 'n'. Remember to make sure that the rows, columns and diagonals cancel out when added. Also make sure that the values around 'n' are opposite. Is it possible to make a new working formula?

Magic Squares

8	1	6
3	5	7
4	9	2

Here is our original magic square.
We know that $8 + 1 + 6 = 4 + 9 + 2$,
and that $8 + 3 + 4 = 6 + 7 + 2$.

However you may not realise that;

$$8 \text{ squared} + 1 \text{ squared} + 6 \text{ squared} = 4 \text{ squared} + 9 \text{ squared} + 2 \text{ squared}$$

In the same way the numbers 8, 3, 4 = 6, 7, 2 when squared

Multigrades

Multigrades are sets of numbers where not only their sums are equal but also their squares, and possibly other powers are equal.

Investigation:

Does this work for other magic squares?

- Square all the numbers of a magic square.
- Investigate the totals in the rows, columns and diagonals.
- **What happens if you multiply by other powers?**